

Star Formation and the Hall Effect

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2003 Jun 20

Abstract. The breakdown of flux-freezing in molecular clouds and protostellar discs is usually approximated by ambipolar diffusion at low densities or by resistive diffusion at high densities. Here I discuss an intermediate regime in which the Hall term in the conductivity tensor is significant, and the vector evolution of the magnetic field – and therefore the evolution of the system under consideration – is dramatically altered. Calculations of charged particle abundances in dense gas in molecular clouds and protostellar discs demonstrate that Hall diffusion is important over a surprisingly broad range of conditions.

Keywords: molecular clouds, star formation, accretion discs, instabilities, magnetohydrodynamics

1. Introduction

The magnetic field in molecular clouds provides pressure support against gravity and carries away angular momentum prior to and during the collapse of cloud cores to form stars. Magnetic fields may also drive the evolution of protostellar discs through the magnetorotational instability, dynamo activity, or by magnetic launching and collimation of jets from their surfaces or inner edges.

The diffusion of magnetic field through the weakly ionised molecular gas plays a crucial role, allowing gas to slip through the magnetic field, or vice-versa. It is usually considered using one of two limits: (i) ambipolar diffusion, in which the magnetic field is frozen into the charged species and drifts along with them through the neutrals; and (ii) resistive diffusion, in which charged particles are completely decoupled from the field by collisions with neutrals. Both limits neglect Hall diffusion, which qualitatively changes the evolution of the magnetic field from any given initial configuration and has profound implications for the magnetically-mediated processes associated with star formation.

Here I present some calculations illustrating the relevance of Hall diffusion to molecular clouds and protostellar discs.



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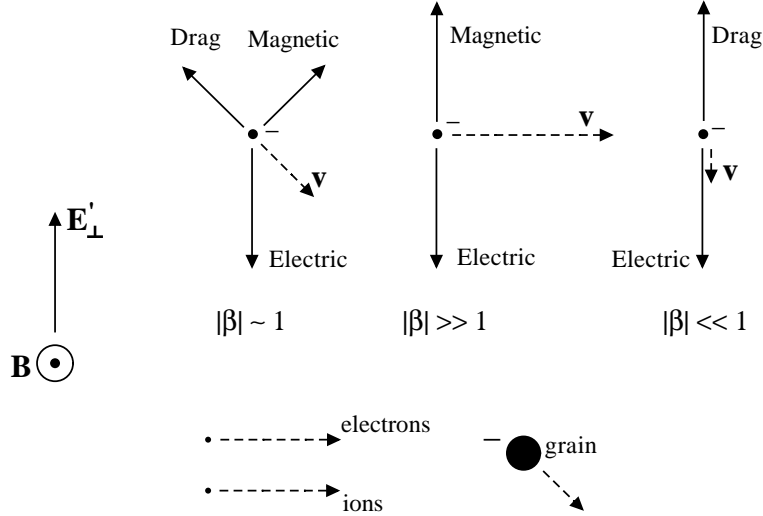


Figure 1. Charged particle drifts in a weakly ionised gas. The upper panel shows the drift speed and force balance for negatively charged particles with different hall parameters (see text). At typical molecular cloud densities ions and electron drifts are set by a balance between the electric and magnetic forces and they drift perpendicular to both the electric and magnetic fields (lower panel). The grain drift is affected by collisions with the neutral particles, the resultant drag causes them to drift at an oblique angle to \mathbf{E}'_{\perp}

2. Hall Diffusion

The diffusion of a magnetic field in weakly ionised gas is determined by the drift of charged particles through the dominant neutral component in response to the electric field in the neutral rest frame, \mathbf{E}' . The drift speed parallel to the magnetic field is set by the drag associated with neutral collisions. In the plane perpendicular to the magnetic field, the electric force $Ze\mathbf{E}'_{\perp}$ on a particle of charge Ze and mass m is balanced by the magnetic and drag forces (see Fig. 1). The relative importance of these two forces is determined by the Hall parameter

$$\beta = \frac{ZeB}{mc} \frac{1}{\gamma\rho}. \quad (1)$$

In the limit $|\beta| \gg 1$, the drag force is negligible, and the drift speed \mathbf{v} satisfies $\mathbf{E}'_{\perp} + \mathbf{v} \times \mathbf{B}/c = 0$. In this case the charged particle is tied to the magnetic field line. In the other limit $|\beta| \ll 1$, the magnetic force is negligible and the drag force $\gamma m \rho \mathbf{v} = Ze\mathbf{E}'_{\perp}$.

At typical molecular cloud densities, ions and electrons have $|\beta| \gg 1$ and drift perpendicular to both \mathbf{B} and \mathbf{E}' . Charged grains, however, have a large collisional cross-section, so that drag is much more

important and $|\beta| \lesssim 1$. Grains therefore drift obliquely to \mathbf{E}'_{\perp} . The grain population acquires a significant charge through the sticking of electrons, and the ensemble of drifting grains modifies the vector relationship between the current density \mathbf{J} and \mathbf{E}' . This relationship can be expressed in terms of a tensor conductivity $\mathbf{J} = \sigma \mathbf{E}'$:

$$\mathbf{J} = \sigma \cdot \mathbf{E}' = \sigma_{\parallel} \mathbf{E}'_{\parallel} + \sigma_{\text{H}} \hat{\mathbf{B}} \times \mathbf{E}'_{\perp} + \sigma_{\text{P}} \mathbf{E}'_{\perp}, \quad (2)$$

where \mathbf{E}'_{\parallel} and \mathbf{E}'_{\perp} are the components of \mathbf{E}' parallel and perpendicular to \mathbf{B} , and σ_{\parallel} , σ_{H} and σ_{P} are the field-parallel, Hall, and Pedersen conductivities, respectively.

The magnetic field evolves according to

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}) - c \nabla \times \mathbf{E}'. \quad (3)$$

The departure from flux-freezing – magnetic diffusion – arises from the second term on the right hand side and is determined by the magnetic field configuration via the conductivity tensor: $\mathbf{E}' = \sigma^{-1} \nabla \times \mathbf{B} / 4\pi$. There are three distinct diffusion regimes:

1. ambipolar diffusion: $|\sigma_{\text{H}}| \ll \sigma_{\text{P}} \ll \sigma_{\parallel}$;
2. hall diffusion: $\sigma_{\text{P}} \lesssim |\sigma_{\text{H}}| \ll \sigma_{\parallel}$; and
3. resistive diffusion: $|\sigma_{\text{H}}| \ll \sigma_{\text{P}} \approx \sigma_{\parallel}$.

The vector evolution of the magnetic field in the presence of Hall diffusion is quite different from the other two regimes. Many of the magnetic field geometries adopted in studies of ambipolar or resistive diffusion – in which the magnetic field lies within a plane of symmetry – break down when Hall drifts are present. The evolution is not invariant under global reversal of the magnetic field and plane-polarised damped Alfvén waves do not exist (Wardle & Ng 1999). The consequences of Hall diffusion during star formation and the subsequent evolution of protostellar discs are profound, as found for example in the magnetorotational instability (Wardle 1999; Balbus & Terquem 2001; Salmeron & Wardle this volume).

3. Charged species and conductivity in molecular clouds

The degree of Hall diffusion in molecular clouds depends on charged particle abundances; especially of grains. Fig. 2 shows the abundances of charged species in molecular clouds for two standard grain models:

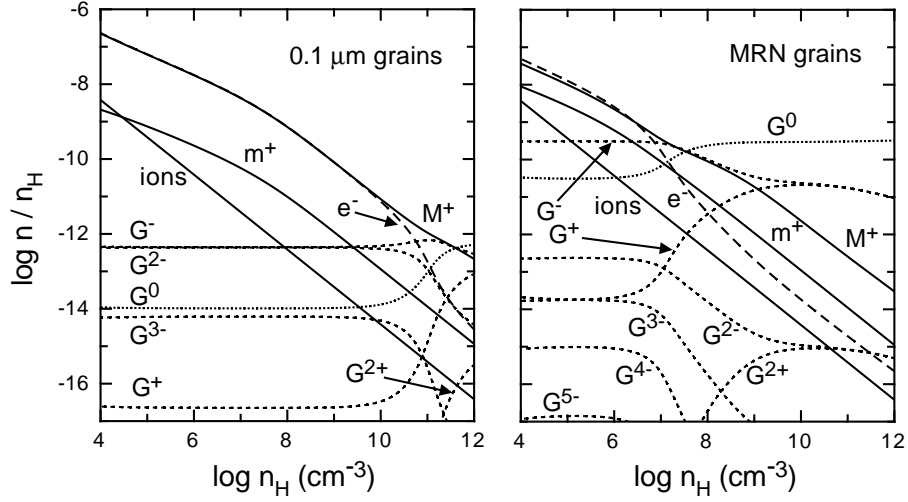


Figure 2. Abundances of charged species in molecular clouds for (left) $0.1 \mu\text{m}$ and (right) MRN grain models.

$0.1 \mu\text{m}$ grains, and grains with an MRN size distribution. These ionisation equilibrium calculations follow Umebayashi & Nakano (1990) and Nishi, Nakano & Umebayashi (1991) in following the abundance of H^+ , H_3^+ , He^+ , C^+ , representative molecular (m^+) and metal (M^+) ions, electrons, and grains. The calculations have been improved by allowing for the higher grain charge states that appear at temperatures in excess of 10 K. Here I have adopted an ionisation rate $\zeta = 10^{-17} \text{ s}^{-1} \text{ H}^{-1}$ and gas temperature 30 K. At this temperature, $0.1 \mu\text{m}$ grains acquire one or two electrons via sticking of electrons. At low densities this has negligible effect on the gas phase ionisation levels because of the relatively small numbers of grains ($n_g/n_H \sim 10^{-12}$). Once the density exceeds 10^{11} cm^{-3} , the sticking of electrons and ions to grains becoming increasingly important, with recombinations occurring predominantly on grain surfaces rather than in the gas phase. An MRN grain size distribution contains many more small grains which typically acquire a single electron. These more numerous grains dominate the recombination process at lower densities ($n_H \gtrsim 10^7 \text{ cm}^{-3}$).

The conductivity tensor for the two grain models can be calculated once the magnetic field is specified. It's instructive to consider the ratio of the Hall and Pedersen conductivities (σ_H and σ_P) as a measure of the significance of the Hall effect in magnetic diffusion. Contours of $|\sigma_H|/\sigma_P$ are plotted in the $B - n_H$ plane for the $0.1 \mu\text{m}$ and MRN grain models in Figs. 3 and 4 respectively. In light of the qualitative changes to magnetic field evolution, I recommend feeling nervous about

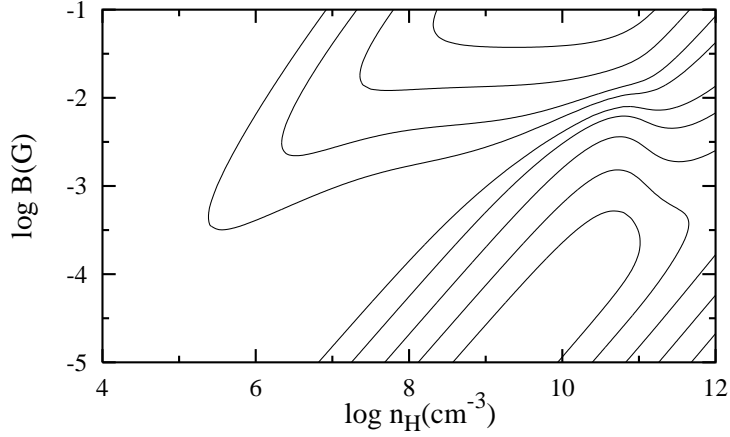


Figure 3. Ratio of the Hall and Pedersen conductivities for the $0.1 \mu\text{m}$ grain model. The contour levels are set at $|\sigma_H|/\sigma_P = 0.01, 0.1, 1, 10$ and 100 .

neglecting Hall diffusion when $|\sigma_H|/\sigma_P$ is above 0.1 , the second contour level in these plots.

For the $0.1 \mu\text{m}$ grain model (Fig. 3), two islands of Hall diffusion are apparent. The left-hand island follows the locus where the grain Hall parameter is of order unity and grains partially decouple from the magnetic field. The island is truncated at low densities where the relative abundance of charged grains becomes too small to influence the conductivity significantly (see Fig. 2). The right-hand island occurs where the ion Hall parameter is of order unity. A zero-line occurs between these two islands where the contributions of grains and ions to σ_H cancel because they carry charge of opposite signs.

The location and size of the grain island depends on the grain size distribution, as size determines both the typical charge acquired by sticking of thermal electrons and ions and the drag coefficient with the neutrals. In Fig. 4, the solid contours show $|\sigma_H|/\sigma_P$ for an MRN size distribution. This has many small grains that acquire a single negative charge (see Fig. 2), so the grain island extends to lower densities but falls off at the highest densities when the abundances of positive and negatively charged grains are almost equal and their Hall contributions cancel. The addition of ice mantles to the grains (dashed contours in Fig. 4) tends to extend the island to the left, as grains have a higher drag coefficient and decouple at lower densities for a given magnetic field strength.

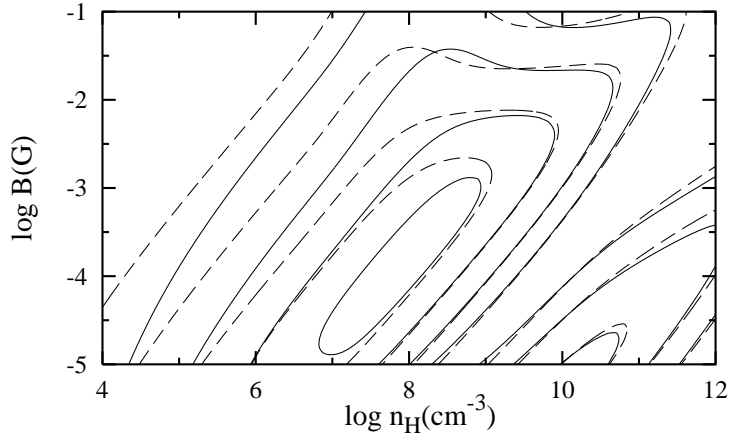


Figure 4. Ratio of the magnitudes of the Hall and Pedersen conductivities for an MRN grain model (*solid contours*) and for MRN grain distribution with ice mantles (*dashed contours*). The contour levels are set at $|\sigma_H|/\sigma_P = 0.01, 0.1, 1$ and 10 .

Clearly, Hall diffusion is significant for both the $0.1 \mu\text{m}$ and MRN grain models at densities $\gtrsim 10^6 \text{ cm}^{-3}$ and the $0.1\text{-}10 \text{ mG}$ magnetic field strengths in molecular clouds. However, the effect is reduced if there are many more small grains (such as PAHs, or an MRN distribution that extends down to a few Angstrom), or if the characteristic grain size is much larger than $0.1 \mu\text{m}$. In the former case, small grains are the dominant charged species at all densities (see Nishi, Nakano & Umebayashi 1991) and the near-equality in abundances of positively- and negatively-charged grains suppresses the Hall effect. In the latter case, grains are sufficiently rare that the grain island is shifted to $\gtrsim 0.1 \text{ G}$, in excess of molecular cloud magnetic field strengths.

4. Conductivity in protostellar discs

Magnetic coupling in protostellar discs is thought to be poor because of two contributing factors. First, the magnetic field must couple to the gas on timescales orders of magnitude shorter than those in molecular clouds. Second, because the gas density is high, recombination occurs more rapidly and the fractional ionisation is low. Third, within a few AU of the central protostar the disc column density shields the gas from cosmic-rays or x-rays. This has led to a picture of magnetic activity occurring only in the surface layers over much of the disc (Gammie 1996; Wardle 1997)

By way of illustration, let us examine the conductivity tensor in a minimum-mass solar nebula model at 1 AU from the central star. The temperature and midplane density are $n_{\text{H}} = 6 \times 10^{14} \text{ cm}^{-3}$ and $T = 280 \text{ K}$ respectively. The disc is assumed to be ionised from above and below by cosmic rays at a rate $10^{-17} \exp(-\Sigma(z)/96 \text{ g cm}^{-2}) \text{ s}^{-1} \text{ H}^{-1}$ where $\Sigma(z)$ is the surface density from height z to the surface, by x-rays from the protostar according to Fig. 3 of Igea & Glassgold (1999) and by natural radioactivity at a uniform rate $10^{-21} \text{ s}^{-1} \text{ H}^{-1}$.

In principle, the grain population is determined by competition between growth or sublimation of ice mantles, sticking, shattering, gravitational settling to the disc midplane, and stirring by convection or turbulence (e.g. Weidenschilling & Cuzzi 1993). For the sake of definiteness, I consider two simple models: (i) $0.1 \text{ } \mu\text{m}$ grains, and (ii) no grains (assuming that they have settled to the midplane). In the latter case, metal atoms play a key role in determining the ionisation fraction in the absence of grains and are conservatively assumed to be depleted by a factor of 10^3 over the interstellar value.

The resulting vertical profile of the conductivity tensor in these two models is plotted in the top panel of Fig. 5 for a uniform 100 mG magnetic field. The much larger conductivities in the no-grain model within three scale heights of the midplane reflects the lack of grains – the charge is carried by more mobile free ions and electrons. The decoupling of ions from the magnetic field for densities $\gtrsim 10^9 \text{ cm}^{-3}$ means that Hall component is larger than the Pedersen component between 1.5 and 4 scale heights, and is 80% of σ_{P} even at the midplane. In the $0.1 \text{ } \mu\text{m}$ grain model, the Hall conductivity dominates the Pedersen conductivity within 4 scale heights of the midplane.

One measure of whether the conductivity is sufficient for the magnetic field to interact with the disc material, is whether the field is unstable to the magnetorotational instability. This is determined by comparing the coupling parameter $\chi = \omega_c/\Omega$ to the ratio of Alfvén speed to sound speed, v_A/c_s . Here ω_c is the frequency above which ideal MHD breaks down and Ω is the Keplerian frequency. If Hall diffusion is unimportant the criterion is $\chi \gtrsim v_A/c_s$, if it is dominant (and the magnetic field has the correct orientation), then $\chi \gtrsim (v_A/c_s)^2$ is necessary.

The coupling parameter is plotted as a function of height in the lower panel of Fig. 5. The entire disc is magnetically active in the settled-grain case, whereas in the single-size grain model the layers above 2.5 scale heights are active. In both cases, Hall diffusion is important throughout the active regions.

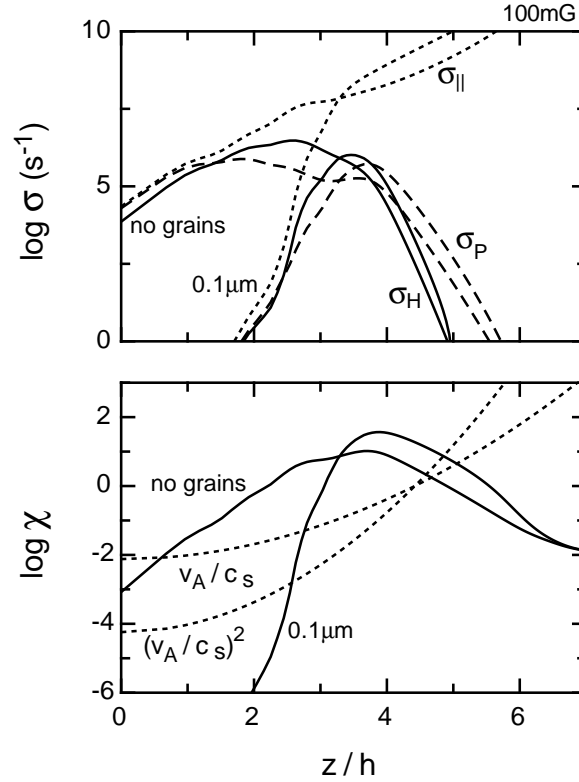


Figure 5. *Upper panel:* conductivity tensor components as a function of height above the midplane at 1AU in a minimum solar nebula for a dusty disc with $0.1 \mu\text{m}$ grains and a model in which grains have stilled to the midplane. *Lower panel:* The vertical profile of the coupling parameter $\chi = \omega_c/\Omega$.

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